Multi-commodity Energy Networks

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Abstract

The Dutch energy grid is increasingly congested, threatening stable future distribution of energy throughout the country. The purpose of this project is to investigate so-called *energy hubs*: small, local nexuses that autonomously control their own energy demand, supply, storage, and conversion. Two use cases are explored independently. The first one involves one energy hub and attempts to optimize cost expectations based on uncertain fluctuations in energy dynamics. The other model describes a network of hubs and explores the limits of using a convex optimization problem.

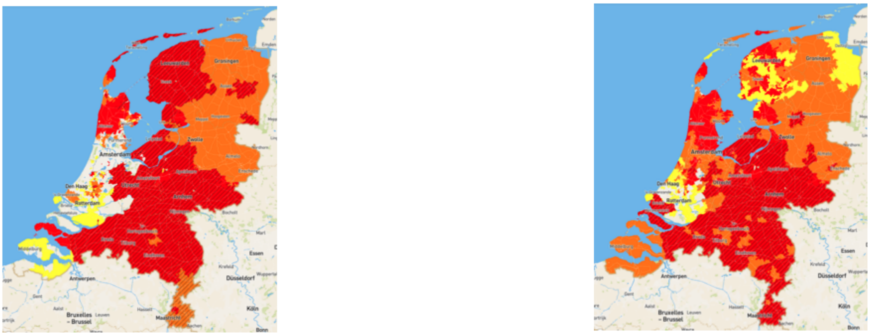
Keywords: Energy hub, multi-commodity, convex optimization, stochasticity.

1 Introduction

The Dutch energy grid is increasingly congested, threatening stable future distribution of energy throughout the country. The situation is illustrated in Figure 1, with limitations on injection of energy in Figure 1a, and on withdrawal in Figure 1b. In both figures, the country is predominantly colored red, indicating no more space for injection and withdrawal, respectively.

The purpose of this project and paper is to investigate so-called *energy hubs*: small, local nexuses that largely control their own energy demand, supply, storage, and conversion. They can vary in, e.g., size, composition, and availability of assets (batteries, solar panels, hydrogen cells, etc). Hubs can be interconnected, enabling them to exchange energy locally and lessening dependency on the main network. The overarching idea of using hubs is that they might help reduce congestion of the main grid by creating smaller, local networks. Potentially, the use of hubs may even facilitate a redesign of the current country-wide main infrastructure.

John Poppelaars, founder of *Doing The Math* (DTM) provided us with the following guidelines. We need to design a model adaptable to evolving energy systems, considering factors like energy flows, investments, carbon emissions, and grid independence supporting decisions on each level. Further, we need to balance model complexity for practical utility, capturing nuances of each level for efficient computation and decision-making. Finally, we aim to explain how the model(s) will provide actionable insights for investors and policymakers in the planning, development, and optimization of multi-commodity energy networks. A discussion among the team revealed that these guidelines required further scoping, as they



(a) Injection limitations (b) Withdrawal limitations

Figure 1: White-yellow-orange-red: in increasing order of congestion. Source: Netbeheer Nederland (2024).

were too broad to tackle in the time frame we had available. In the end, we settled on working out the following scenarios:

1. A *stochastic* model for a hub connecting companies that are supplied with wind and solar power, electricity from the grid, and hydrogen via a pipeline. Additionally, they have a shared battery and shared storage for hydrogen. The main goal was to find out to what extent dynamic control of energy buying and selling may reduce the costs of energy, as compared to static control. This is covered in Section 2.

2. A convex model for a hub with wind and solar power, unlimited electricity from the grid, and a battery. We discuss this in Section 3.1. We also consider the finite access to electricity in Section 3.2.

3. A convex model as in Section 2, but with several hubs (Section 3.3).

During our initial discussion, we had several promising ideas for potential future projects. We collected these in Section 4.

2 Stochastic electricity and hydrogen model

We consider a hub with companies that are supplied by 1) wind and solar power, 2) electricity from the grid, and 3) hydrogen via a pipeline. For energy management purposes, the companies share a battery in order to provide power in times when electricity from the grid is relatively expensive (due to high grid load, e.g., in the middle of the day). Furthermore, the companies share a hydrogen storage tank to supply hydrogen as a substitute for natural gas or to convert hydrogen into power through an electrochemical reaction known as *reverse electrolysis*. The battery can be filled in times when wind and solar power supply exceeds demand or when grid electricity is relatively cheap. The dynamics of the battery status, from hour to hour, is modeled as

$$x_{k+1}^b = x_k^b + \Delta t(u_{1,k} - u_{2,k} + \beta_1\gamma_3 u_{3,k} - u_{4,k} + s_k - d_k - \alpha_k) + \sigma\sqrt{\Delta t}\varepsilon_k. \tag{1}$$

Here $k \in [1, \ldots, N]$ is an index ranging from hour 1 to hour N (the time interval corresponding to $k = 1$ is 00.00 to 01.00 hours), Δt the time interval, x^b the battery status which is bounded by a minimum and a maximum ($x_k^b \in [b^{\min}, b^{\max}]$), $u_1 \in [0, u_1^{\max}]$ the power bought from the grid, $u_2 \in [0, u_2^{\max}]$ the power sold to the grid, $u_3 \in [0, u_3^{\max}]$ the hydrogen converted to power with efficiency β_1, u_4 conversion of power to hydrogen, s the supply from wind and solar power, d the power demand from the companies, α energy leakage from the battery. The uncertain error ε represents the difference between predicted and realized battery status due to uncertainty in demand and supply, and this error is assumed to be independently normally distributed. This is a standard assumption in stochastic optimization, although another type of distribution can also be inserted. The error is scaled with variance σ^2. The hydrogen level dynamics are modeled as

$$x_{k+1}^H = x_k^H + \Delta t(\beta_2(x_{k+1}^b - b^{\max} - \gamma_4 u_{4,k}) - u_{3,k} + u_{5,k} - u_{6,k}). \tag{2}$$

Here x^H is the hydrogen level which is bounded by a minimum and a maximum ($x_k^H \in [H^{\min}, H^{\max}]$). The term $x_{k+1}^b - b_{\max}$ represents the overflow rate of the battery that is converted into hydrogen with efficiency β_2. Furthermore, u_5 denotes the hydrogen buying rate, and u_6 denotes the selling rate.

2.1 Costs and revenues

The expectancy of the total costs that should be minimized is modeled as

$$\mathbb{E}\left[-c_1 x_N^b - c_5 x_N^H + \Delta t \sum_{k=1}^{N-1} c_{1,k}u_{1,k} - c_{2,k}u_{2,k} + c_{5,k}u_{5,k} - c_{6,k}u_{6,k}\right]. \tag{3}$$

The first two terms denote the revenues associated with having the power and hydrogen in store at the end of the day. The terms behind the summation denote running costs that are made each hour. Here, the c values denote the buying and selling prices of electricity and hydrogen.

The symbols and nominal values used are presented in Table 1.

2.2 Dynamic programming

The goal is to determine a policy that maximizes value, which is defined by the expected revenues $J(x_N^g)$ for a battery at end time $N\Delta t$, minus the compounded running costs L_k. The associated control problem is to find a policy that minimizes expression (3), which can be written as

$$\mathbb{E}\left[J(x_N^g) - \sum_{k=0}^{N-1} L_k(g_k(x_k^g))\Delta t\right]. \tag{4}$$

Symbol	Description	Value(s)	Unit
c_1	price electricity bought from grid	1	[euro kWh^{-1}]
c_2	revenues electricity sold to grid	0.8	[euro kWh^{-1}]
c_5	price hydrogen bought	1	[euro kg^{-1}]
c_6	price hydrogen sold	0.8	[euro kg^{-1}]
c_7	costs associated with unmet power demand	100	[euro kW^{-1}]
s	supply rate from PV and turbines	[dataset]	[kW]
d	demand from companies	[dataset]	[kW]
$u_1 \in [0, u_1^{\max}]$	electricity buying rate from grid	[0,2]	[kW]
$u_2 \in [0, u_2^{\max}]$	electricity selling rate to grid	[0,2]	[kW]
$u_3 \in [0, u_3^{\max}]$	hydrogen conversion to power	-	[kWh kg^{-1}]
$u_4 \in [0, u_4^{\max}]$	power conversion to hydrogen	-	[kWh^{-1} kg]
$u_5 \in [0, u_5^{\max}]$	hydrogen buying rate	-	[kg^{-1}h^{-1}]
$u_6 \in [0, u_6^{\max}]$	hydrogen selling rate	-	[kg^{-1}h^{-1}]
N	number of hours in simulation	24	[h]
$x^b \in [b^{\min}, b^{\max}]$	battery level	[1,24]	[kWh]
$x^H \in [H^{\min}, H^{\max}]$	hydrogen level	-	[kg]
α	battery leakage rate	0.001	[kW h^{-1}]
β_1	efficiency conversion hydrogen to power	-	[-]
β_2	efficiency conversion power to hydrogen	-	[-]
γ_3	conversion coefficient hydrogen to power	-	[kWh kg^{-1}]
γ_4	conversion coefficient power to hydrogen	-	[kg kWh^{-1}]
σ	state noise standard deviation	0.003	[kW]
Δt	time interval	1	[h]

Table 1: Symbols used in the electricity hub model, together with their units and values used in the simulations.

for any given state x_k, at any time k, over all control laws $(g_k)_{k\in\mathbb{K}}$ such that g_k is in an admissible set $\mathcal{G}_k$ (defined in Table 1 as $[0, u_{(\cdot)}^{\max}]$). Here x^g denotes $[x^b, x^H]$ under policy g. We use $\mathcal{G}$ to denote the collection of admissible sets $(\mathcal{G}_k)_{k\in\mathbb{K}}$.

2.3 Solution of optimization problem for only electricity

We consider the solution of equation 4 for a hub only regarding power management, ignoring the hydrogen aspects and taking $\Delta t = 1$ hour. By doing so, equation (1) reduces to

$$x_{k+1}^b = x_k^b + u_{1,k} - u_{2,k} + s_k - d_k - \alpha_k + \sigma\sqrt{\Delta t}\varepsilon_k. \tag{5}$$

We define the following constraint in order to avoid negative battery charge, $\mathbb{E}x_{k+1}^b \geq b^{\min}$, which translates into $u_{1,k} \geq b^{\min} - x_k + u_{2,k} - d_k + d_k + \alpha_k$.

The expectancy of the total costs (3) reduces to

$$\mathbb{E}\left[-c_1 x_N^b + \Delta t \sum_{k=1}^{N-1} c_{1,k}u_{1,k} - c_{2,k}u_{2,k}\right]. \tag{6}$$

The peak prices in electricity during 12.00-18.00 hours are modeled as

$$c_{1,k} = \mathbf{1}_{k\in[1,12]\oplus[19,24]} + 2\,\mathbf{1}_{k\in[13,18]}, \tag{7}$$

and furthermore, the selling price is assumed to be 80% of the buying price: $c_{2,k} = 0.8c_{1,k}\ \forall k$.

The control problem of minimizing (6) was solved using a dynamic programming method as described in van Mourik et al. (2023). The state space was made discrete into 300 parts. The dynamics of d and s was an hourly time series data set that was provided by the DTM company.

2.4 Results

Figure 2 shows the dynamics of battery level, resulting from arbitrarily selected constant buying and selling rates and supply and demand dynamics. In this case, the buying and selling rates are not optimized. It can be observed that the battery empties over time from 24 to 17 KWh. This is due to demand being higher than supply plus the net difference between buying and selling.

Figure 3 shows the dynamics of battery level resulting from constant buying and selling rates optimized with respect to the cost function (6). It is observed that the buying and selling rates are very close to zero, and as a result, the battery is much more depleted compared to Figure 2.

Figure 4 shows the optimal dynamic control of buying and selling rates as a function of the battery state and time with respect to cost function (6).

It is observed that there is a vertical band between 12 and 18 hours in which there is practically no buying due to high electricity prices. For the same reason, the controller prescribes a maximum selling rate during these hours, except when the battery is nearly empty. The expected running costs for the three cases described above (arbitrarily selected constant input,

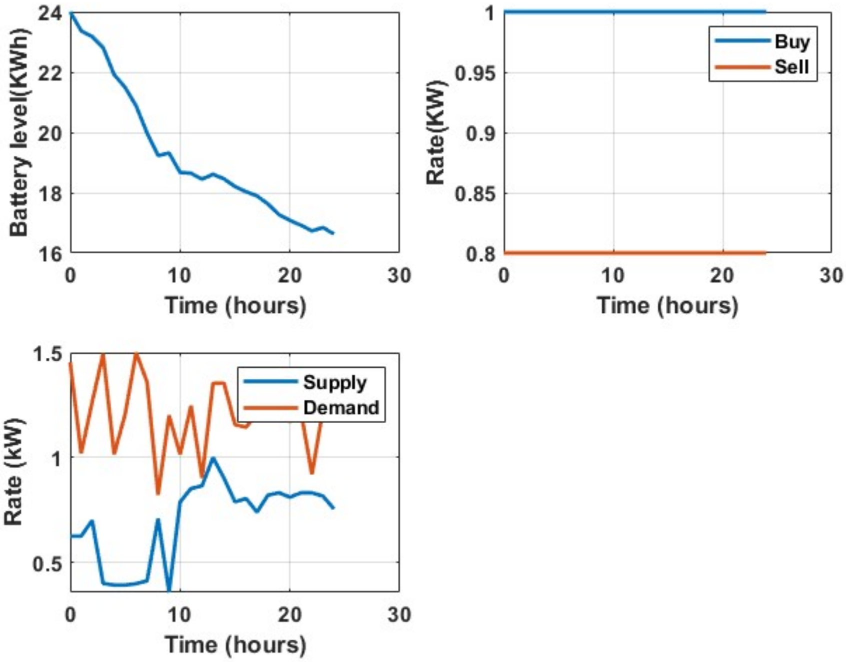


Figure 2: Dynamics of battery level for arbitrarily selected constant buying and selling rates. Top left: battery level. Top right: constant buying and selling rates. Bottom left: supply and demand rates.

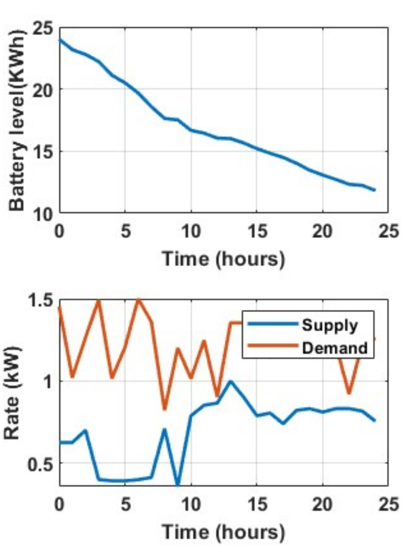


Figure 3: Dynamics of battery level for optimal constant buying and selling rates. Left top: battery level. Right top: constant buying and selling rates. Left bottom: supply and demand rates.

optimized constant input, and optimized dynamic input) are 18, 12, and 6 Euro, respectively. The large relative differences amounts indicate that dynamic optimization might considerably improve cost efficiency in the daily operational management of an energy hub.

3 Convex Energy hub model

Another model that does not incorporate a stochastic approach is laid down here. The idea behind this model relies on the neat properties of convex functions and linear programming. Different scenarios depending on the type of hub that we want to create are discussed here.

3.1 Infinite supply of energy from provider

Suppose that the contract with the energy provider allows for very large amounts of energy to be taken out of the grid. Thus, we allow for very large $u_{1,k}$. This assumption is posed in order to guarantee that no matter how big the desired outcome is, the production is able to follow. In the next Section 3.2, we will drop this assumption and consider the more likely scenario of a capped subscription to the energy grid. Still, it is important to consider this "infinite" energy provider case as it can apply to multiple scenarios, including those with companies that are mostly energy-autonomous but which, once in a while, might need some unknown energy input from the grid.

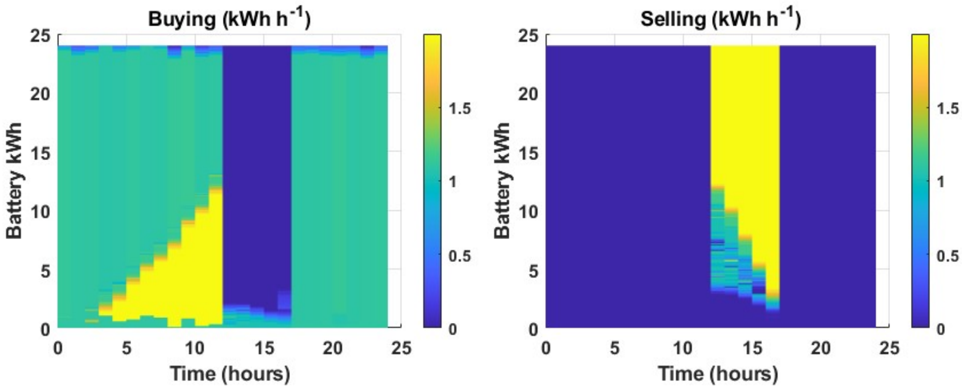


Figure 4: Optimal dynamic control of buying and selling rates as a function of the battery state and time. Left: buying rate as a function of time and state. Right: selling rate as a function of time and state.

We start by using the same symbols as in Section 2. The desired cost function must be convex and must minimize the costs associated with using the provider's electricity while introducing a penalty for using too large quantities of the provider's supply.

Problem formulation:

for $n = 1, \dots, K$

$$\min_{u_{1,k}} \sum_{k=n}^{K} A(u_{1,k}), \tag{8}$$

where A is a penalizing function for large u_1. The constraints on the problem are

$$\begin{cases} w_k + x^b_{k+1} \le x^b_k + s_k - d_k + u_{1,k} \\ x^H_{k+1} = x^H_k + \beta w_k \end{cases} \quad \text{with} \begin{cases} 0 \le u_1 \le u_1^{\max} \\ 0 \le x^b \le b^{\max} \\ 0 \le w \\ 0 \le x^H \le H^{\max} \\ 0 \le d \end{cases} \tag{9}$$

where w_k is the electricity quantity going from or to the hydrogen plant. Further, β reflects the conversion factor from electricity to hydrogen.

Keeping the problem convex

As long as we choose A to be convex, the problem is itself convex. Examples of a convex penalising function is the exponential function, which conveniently increases very rapidly.

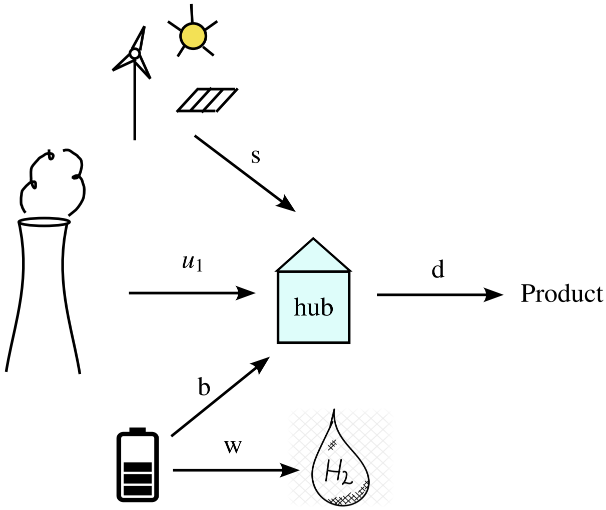


Figure 5: Diagram of basic relationships between hub components. Clockwise from the battery icon, we have the state's grid, the local power supply such as wind and PV, the end product, and a storage component such as a battery.

3.2 Finite supply of energy

for $n = 1, ...K$

$$\max_{u_{1,k}} \sum_{k=n}^{K} \chi_k d_k - \sum_{k=n}^{K} c_{1,k} u_{1,k}, \tag{10}$$

where the linear relationship can be replaced with a convex one. The constraints on the problem are

$$\begin{cases} w_k + x_{k+1}^b \leq x_k^b + s_k - d_k + u_{1,k} \\ x_{k+1}^H = x_k^H + \beta w_k \end{cases} \quad \text{with} \begin{cases} 0 \leq u_1 \\ 0 \leq x^b \leq b^{\max} \\ 0 \leq w \\ 0 \leq x^H \leq H^{\max} \\ 0 \leq d \end{cases} \tag{11}$$

Note: The reason why the above inequality relating w, x^b, s, d and u_1 is not strict is that the battery has finite capacity and excess supply goes unused (this is called *curtailment*. This is in the hope that the excess can be managed in a number of different ways. Examples include the selling of surplus, as well as the production of other byproducts (carbon sequestration, hydrogen production through electrolysis, etc.). An equality would mean that g_k can become negative, allowing electricity to be sold back to the main network.

Discussion on the choice of functions

We have assumed here a linear behavior for the revenue (e.g., $\chi_k d_k$), but perhaps the relationship is more complex. This is no problem as long as we choose a concave function. We theorize here that another function could be chosen, such as $\cot(\cdot)$ or $\sqrt{\cdot}$. These are concave, and their rate of increase is not high. This resonates with most business practices (the production curve will plateau after some production threshold).

3.3 Introducing several hubs

Extending the model to incorporate different hubs can be done precautiously. The following is a model that connects different hubs with indices $i \in \mathcal{I}$ possibly with cables in between each other, which nonetheless demands some investments as cables are expensive. In this way, surplus energy could be shared without relying extensively on the external grid. To this end, a common battery is required to act as the main nexus of electricity exchange. As batteries are expensive, this incentivizes different hubs to invest jointly in one battery that is consequently installed on one of the hubs' sites. It should be accessible to the other hubs so that they can store or access surplus energy before resorting to the grid. A diagram showing how different hubs intertwine is shown in Figure 6.

To model this, we again attempt to create a convex problem. We tweak the original notation slightly and denote δ_k^i to be the amount of electricity that hub i asks at time k to have. Further, denote p_k^i to be the amount of energy surplus that hub i generates and has no need of. We want to maximize the gain so that the following constraints are satisfied:

$$\begin{aligned} s_k^i + u_{1,k}^i + \delta_k^i &= d_k^i + p_k^i \qquad\qquad i \in \mathcal{I}, \\ x_{k+1}^b &= x_k^b - \sum_i \delta_k^i + \sum_i p_k^i. \end{aligned}$$

Now, finding the revenue/cost function to maximize is harder as we have to encompass multiple hubs. A simple approach that we put forward here is maximizing the sum of revenue-cost for each hub, e.g.,

$$\max_{u_{1,k}^i} \sum_i \sum_{k=n}^{K} \chi_k d_k^i - \sum_{k=n}^{K} c_k u_{1,k}^i. \tag{12}$$

Upon seeing this formulation, one might think that this shared-battery model is suboptimal to each having their own batteries. Indeed, it is well known that the maximum of a sum is smaller than the sum of maxima. Equation 13 then would yield a result less than or equal to the sum of each maximization. In the case of $i = 1, 2$,

$$\max_{u_{1,k}^i} \sum_{i=1}^{2} \sum_{k=n}^{K} \chi_k d_k^i - \sum_{k=n}^{K} c_k u_{1,k}^i \leq \sum_{i=1}^{2} \left(\max_{u_{1,k}^i} \sum_{k=n}^{K} \chi_k d_k^i - \sum_{k=n}^{K} c_k u_{1,k}^i \right). \tag{13}$$

The question arises: Why not maximize each hub individually again and then sum to obtain the gain? The answer is that the $u_{1,k}^i$ over which we optimize are dependent on how much

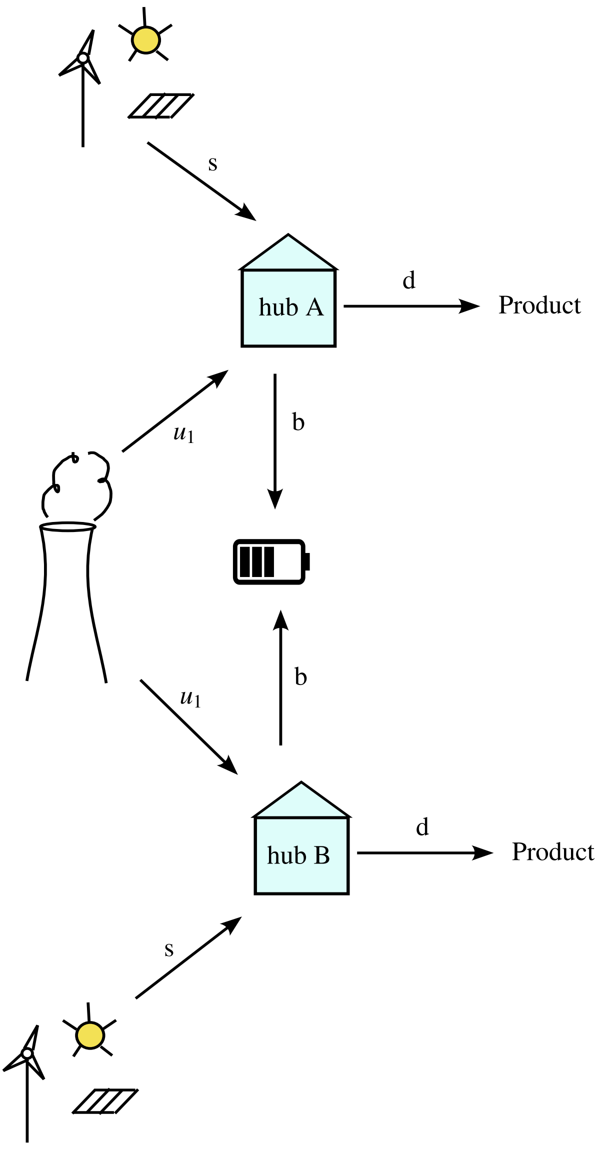


Figure 6: Diagram of basic relationships between two hubs and their components.

energy δ_k^i they can withdraw from the common battery. Evidently, we cannot optimize each hub individually without taking into account the other hubs.

Note: A natural question comes into mind. What if, in a non-collaborative setting (e.g., each hub has its own battery), one hub makes more than in a collaborative setting? How do we prevent unfair scenarios in which collaborating helps some hubs more than others? To make it concrete, consider a non-collaborative scenario (call it Scenario 1) in which hub A makes $100e$ profit and hub B makes $10e$ profit. The difference in profit might be due to a bigger energy park within hub A or a bigger battery capacity. Suppose that in a collaborative scenario, where batteries are shared (call it Scenario 2), hub A makes $90e$ profit whilst hub B makes $30e$. There is indeed an *overall* gain to have collaborated, in the sense that added together, the gain is $120e$ instead of $110e$. But hub A has not benefited from having collaborated. This type of scenario requires compensation towards hubs that would have received more if they had functioned alone. A simple suggestion is that the profit from having collaborated should be split proportionately to the participation of each hub. Practically, it could be done by looking at the overall profit in Scenario 1 and in Scenario 2 and splitting the difference in overall gain so that even each hub obtains at least what they would have obtained if they had functioned alone. In this example, let hub A receive at least $100e$, which means hub B obtains at most $20e$, and the remaining surplus can be split further between the two parties. In this way, each hub receives more than if they had functioned alone.

Nevertheless, how can we be certain that collaborating would lead to an increase in wealth and not to a decrease? The intuitive answer to this is that instead of buying from the grid, one can use from the battery and pay smaller costs than when using from the grid. It seems intuitive that it is beneficial, but it needs more formal proof of it. For example, what if it costs more to produce local energy than to use the grid? Then, perhaps each hub will preferentially use the grid, in which case, in the worst case, the gain will be the same as in Scenario 1. Therefore, we need to make sure that the costs associated with the running and maintenance of the local energy hubs warrant their existence–future work needs to incorporate weigh-in data regarding the overall costs of running a local nexus.

4 Conclusions

As it stands, this paper has developed two energy models that could help direct future companies in making decisions that can have a major economic as well as environmental impact. For the model of stochastic electricity demand and the basic scenario where electricity can be stored, bought, or sold, the expected net electricity costs for three cases (arbitrarily selected constant input, optimized constant input, and optimized dynamic input) are 18, 12, and 6 Euro, respectively. The large relative differences amounts indicate that dynamic optimization might considerably improve cost efficiency in the daily operational management of an energy hub. Of course, at this point, the values are not yet realistic, but the case could form a starting point for a more in-depth analysis in a follow-up study.

This work focused on modeling rather than simulation, mainly because it would allow a better grasp of the real stakes. Therefore, further work could be done to investigate the second model through a simulation study and see how it compares to the first model. Furthermore, future research should be directed to combining both models in order to bring stochasticity into the multi-commodity and multi-hub model. Lastly, we restricted our work to energy hubs that have a predetermined set of assets (e.g., the number of solar panels and windmills). Future research can explore which combinations of assets are optimal given the local average weather and climate, as well as the desired production rates.

The research done during the SWI event raised the interest of Doing The Math to further explore the directions taken and see if the suggestions for further research could be turned into, for example, thesis subjects. Next, applying the ideas in this paper to real-life situations with actual data could lead to new insights from which distribution system operators could benefit as it will direct their choices to enable energy hubs and reduce the impact of congestion on society.

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